Mass Transfer in Eccentric Binary Stars

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Abstract

The concept of Roche lobe overflow is fundamental to the theory of interacting binaries. Based on potential theory, it is dependent on all the relevant material corotating in a single frame of reference. Therefore if the mass losing star is asynchronous with the orbital motion or the orbit is eccentric the simple theory no longer applies and no exact analytical treatment has been found. We use an analytic approximation whose predictions are largely justified by SPH simulations. We present smoothed-particle-hydrodynamics simulations of binary systems with the same semi-major axis $a = 5.55\,R_\odot$, masses $M_1 = 1\,M_\odot$, $M_2 = 2\,M_\odot$ and radius $R_1 = 0.89\,R_\odot$ for the primary star but with different eccentricities of $e = 0.4$, 0.5, 0.6 and 0.7. In each case the secondary star is treated as a point mass. When $e = 0.4$ no mass is lost from the primary while at $e = 0.7$ catastrophic mass transfer, partly through the L_2 point takes place near periastron. This would probably lead to common-envelope evolution if star 1 were a giant or to coalescence for a main-sequence star. In between, at $e \geq 0.5$, some mass is lost through the L_1 point from the primary close to periastron. However, rather than being all accreted by the secondary some of the stream appears to leave the system. Our results indicate that the radius of the Roche lobe is similar to circular binaries when calculated for the separation and angular velocity at periastron. Part of the mass-loss occurs through the L_2 point.

Key words: gravitation, hydrodynamics, stars: binaries, stars: evolution, stars: mass-loss, X-rays: binaries.

1 Introduction

The theory of tidal interaction in binary systems, both stellar and planetary, has been the topic of much research as has the interaction, by mass transfer, of close binaries. Tidal forces set up oscillating displacements in the two objects and the loss of energy through the damping of these displacements, viscously or otherwise, leads to the circularization of the orbit and the synchronization of the spins of the objects with their orbital motion. However, tidal interaction theory is not fully understood in two important aspects.

First, although since Press & Teukolsky (1977) the nature of the oscillations has been understood, various alternative theories exist for the viscous damping mechanisms. For instance in the case of convective envelopes damping can take place viscously as momentum is transported by convective cells but precisely how is still in dispute (Zahn 1977, Goldreich & Keeley 1977, Tassoul & Tassoul 1992). New observational evidence can be used to resolve some of these uncertainties (Leibowitz & Ofek 1997).

Second, formulae for circularization time scales based on approximations for near-circular orbits are often used, without care, by investigators working on interacting binary stars. This situation has been somewhat remedied by the work of Mardling (1995) who has examined the chaotic growth of oscillations over many orbits. The intricate relation between excitation and damping of various modes of oscillation has still to be resolved fully in the case of eccentric orbits. In the past this has not been of very great importance because binary stars that interact usually reach the point of interaction by relatively slow nuclear evolution during which tidal forces have had ample time to both circularize the orbit and synchronize at least the most unstable star and so evolution proceeds independently of the tidal history.

Two rapidly growing fields of research now serve to make the understanding of tides in eccentric systems much more important. First, detailed models, particularly of an N-body nature, of stellar clusters, both open and globular, have been constructed including stellar evolution and binary star interaction of the individual components (Tout et al. 1997, Mardling & Aarseth 2001). Within these clusters processes such as close encounters and tidal capture can lead to systems that will undergo mass transfer while their orbits are still eccentric. Capture processes are particularly important for the formation and subsequent evolution of X-ray binaries, the distribution of whose properties serves as an important constraint on any theory. In addition some eccentric binaries are actually observed undergoing mass transfer (Belloni et al. 1996) although this may be predominantly via a stellar wind.

Also observations of complete samples of main-sequence and younger binary systems reveal an intriguing distribution of eccentricities (Mathieu 1994). As expected the closest systems are all circular but above a critical period the observed eccentricities apparently uniformly fill an envelope giving a maximum eccentricity for a given period which grows with period. It is claimed that it is not possible to explain this envelope in terms of tidal circularization and hence it is believed to contain important information about the formation of binary stars (Duquennoy & Mayor 1991). The CORAVEL radial velocity surveys in the solar neighbourhood, Pleiades and Praesepe confirm that the period – eccentricity diagrams are consistent among field stars and the binaries in open clusters (Halbwachs et al. 2003). The relationship of eccentricity to period indicates the tidal circularization cutoff for the cluster. Various recent surveys of proper motion stars (Latham et al. 1988, Boffin et al. 1993, Mazeh et al. 1997, Carney et al. 2001, Latham et al. 2002, Tokovinin et al. 2003, Mathieu et al. 2004) and MACHO project eclipsing binaries (Alcock 2003) have revealed aspects of the orbital eccentricity distribution, however, in the light of the above, tidal circularization mechanisms have not been extensively tested. Such an investigation represents an interesting future development.

Below we present two approaches to the problem of mass transfer in eccentric binary stars. In Section 2 we outline the fundamental differences between circular synchronized and eccentric

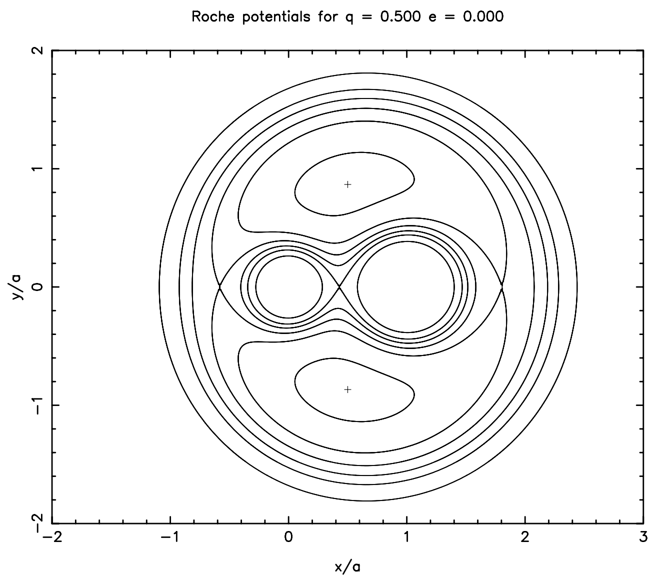

Figure 1: A section through the Roche equipotential surfaces for mass ratio $q = 0.5$ in the orbital plane. The surfaces through the L_1, L_2 and L_3 points with intersections on the x axis are included.

systems. In Section 3 we calculate the actual forces on the material at the surface of the donor star, taking into account its structure and the potential of the companion in a frame corotating with its own spin. At periastron this determines an effective Roche lobe radius (such that for radii less than this no mass transfer takes place) by ensuring that nowhere does the resulting force point away from the star. In Section 4 we present SPH simulations of eccentric asynchronous binaries. Section 5 is our Conclusions.

2 Mass Transfer in Eccentric Binaries

Leading up to and since the establishment of the solution to the Algol paradox (Hoyle 1955, Crawford 1955) much work has been devoted to the study of mass transfer in synchronized circular binaries. However, relatively little has been done for the case of eccentric binaries. The usual approach has been to use either the standard Roche-lobe theory or some variant of it, such as calculating an effective Roche lobe at periastron. The Roche lobe is the last stable equipotential surface to which a star can grow, in a circular orbit, before its surface material is more attracted to its companion than to itself.

In circular, synchronized systems the rate of stable mass transfer is a very strong function of the amount by which the star overfills its Roche lobe. The rate adjusts on a dynamical timescale until a balance is achieved in which the star only just overfills its Roche lobe and the mass transfer rate is easily computed by finding the rate necessary to keep the star just filling its Roche lobe. Since the shape of this equipotential surface is not far from a sphere a very useful approximation to the evolution can be achieved by defining a Roche lobe radius R_{L}, proportional to separation, which is the radius of a sphere with the same volume as the lobe. The evolution of the star can then be adequately computed by considering it as a single spherical star as long as its radius $R < R_{\mathrm{L}}$. If $R > R_{\mathrm{L}}$ mass transfer from its surface proceeds at a rate that ensures $R \approx R_{\mathrm{L}}$. An accurate empirical fit to the Roche-lobe radius of star 1 is

$$\frac{R_{\mathrm{L}}}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})}, \tag{1}$$

where the mass ratio $q = M_1/M_2$ and semi-major axis a (Eggleton 1983).

The simple Roche-lobe potential theory is not tenable in eccentric binaries because it relies on the elimination of Coriolis' forces in a single rotating frame in which all the material is corotating. Such a frame does not exist in eccentric or even asynchronous binaries.

Two objectives are immediately apparent. First to determine an effective Roche-lobe radius for eccentric asynchronous binaries $R_{\mathrm{L}}(e, \omega, a)$ such that for radii less than this no mass transfer takes place, for eccentricity e and stellar spin rate ω. Second to determine the rate of mass transfer as a function of R and $R_{\mathrm{L}}(e, \omega, a)$ when the radius is greater than this.

For the circular synchronized case, when a star grows faster than its Roche lobe on losing mass, mass transfer is unstable. In the circular synchronized state when a deeply convective star begins to overflow it grows so that the mass transfer increases on a dynamical time scale. This positive feedback leads to common-envelope evolution (Paczyński 1976), a necessary step in creating very close binaries with degenerate components from binary evolution alone. The aim here is to determine under what conditions such dynamic evolution follows the onset of mass transfer in eccentric systems. The situation will differ significantly from the circular case because at its onset mass transfer will be episodic (occurring only near periastron) and mass transfer itself will alter the eccentricity as well as the separation.

In this paper we discuss two approaches to the problem. The first is to consider an approximation to the acceleration equations, taking into account the inertial forces, that allows us to describe a set of Roche-like potential surfaces at periastron. Comparison of these with the circular, synchronous state yields qualitative insight into what might be expected in general. We then present smoothed-particle-hydrodynamic simulations of various eccentric systems that transfer mass during periastron passage.

3 Approximate Potential at Periastron Passage

Consider a binary system with eccentricity e consisting of a point-like star 2 of mass M_2 and a centrally condensed but extended star 1 of mass M_1 and radius R spinning with angular speed ω about an axis perpendicular to the orbital plane. The angular velocity $\mathbf{\Omega_p}$ of the orbit at periastron has a magnitude given by

$$\Omega_{\mathrm{p}}^2 = \frac{G(M_1 + M_2)}{r_{\mathrm{p}}^3}(1 + e), \tag{2}$$

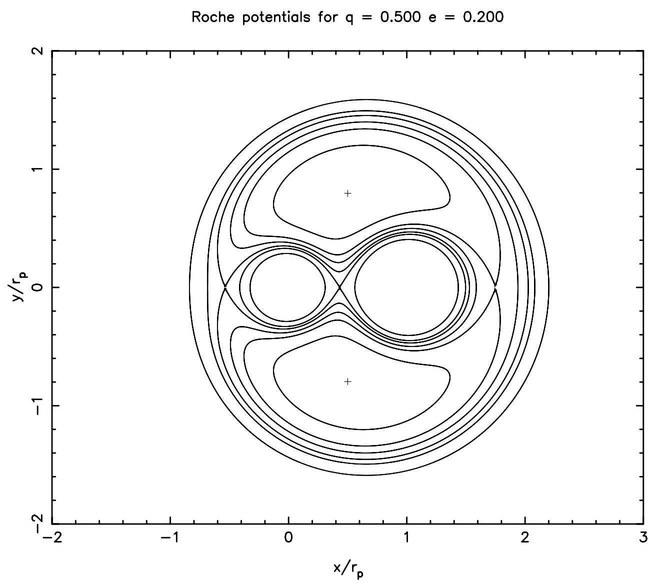

Figure 2: Pseudopotential surfaces for $q = 0.5$ and eccentricity $e = 0.2$ at periastron passage. The surfaces through the pseudo-L_1, pseudo-L_2 and pseudo-L_3 points with intersections on the x axis are included.

where G is the gravitational constant and

$$r_\mathrm{p} = a(1 - e) \tag{3}$$

is the periastron separation for an orbit of semi-major axis a. Now consider a rotating coordinate system centred on star 1 and rotating at $\mathbf{\Omega_p}$. Tides tend to locally synchronize (pseudosynchronize) the spin of the extended star such that

$$\omega \leq 1.25\Omega_\mathrm{p} \tag{4}$$

(Hut 1981) so that we are not far off in making the approximation $\omega = \Omega_\mathrm{p}$. The system is then instantaneously stationary in the rotating frame.

The total acceleration of a particle with position vector $\mathbf{r}$ in this rotating frame is (omitting the acceleration of the centre-of-mass and $\dot{\mathbf{\Omega}}_\mathbf{p} \wedge \mathbf{r}$ which are $\mathbf{0}$)

$$\mathbf{a} = \ddot{\mathbf{r}} + 2\mathbf{\Omega_p} \wedge \dot{\mathbf{r}} + \mathbf{\Omega_p} \wedge (\mathbf{\Omega_p} \wedge \mathbf{r}). \tag{5}$$

At periastron $\dot{\mathbf{r}} = \mathbf{0}$. A strict condition for no mass transfer to occur would be to require that nowhere does the resulting force point away from the star, i.e. $\ddot{\mathbf{r}}.\mathbf{n} \leq 0$ everywhere, where $\mathbf{n}$ is the normal vector to the surface. We proceed by making the approximation $\ddot{\mathbf{r}} = \mathbf{0}$. Then, as in the circular synchronous state (see Pringle 1985), we may write the Euler equation

$$\mathbf{a} = -\frac{1}{\rho}\nabla p - \nabla\phi_\mathrm{G} - \nabla\phi_\Omega, \tag{6}$$

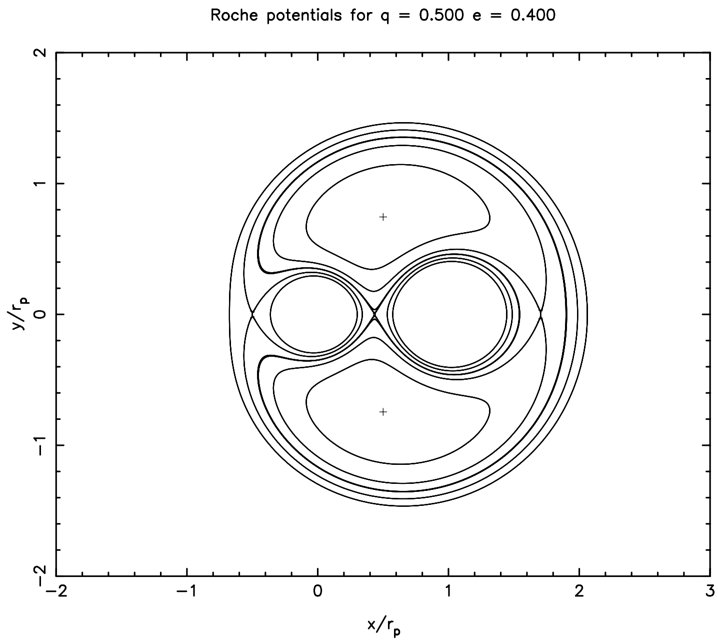

Figure 3: As Figure 2 but with $e = 0.4$.

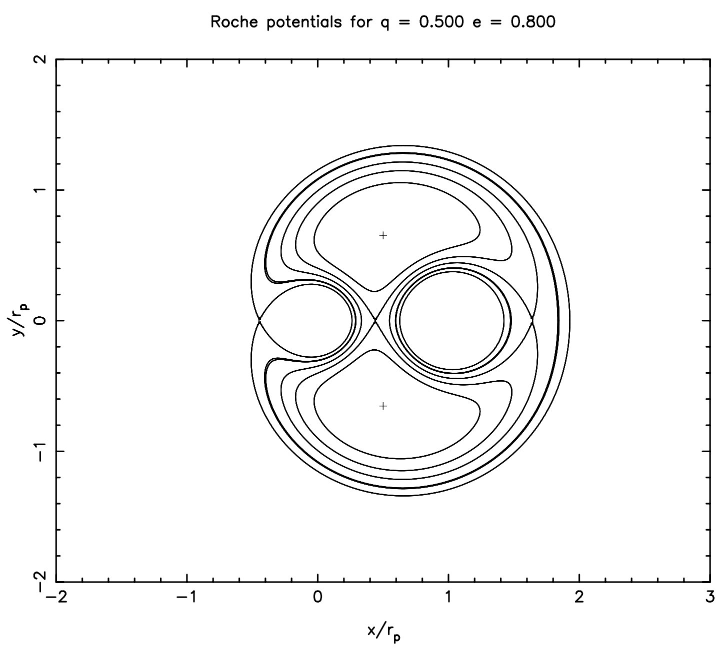

Figure 4: As Figure 2 but with $e = 0.8$.

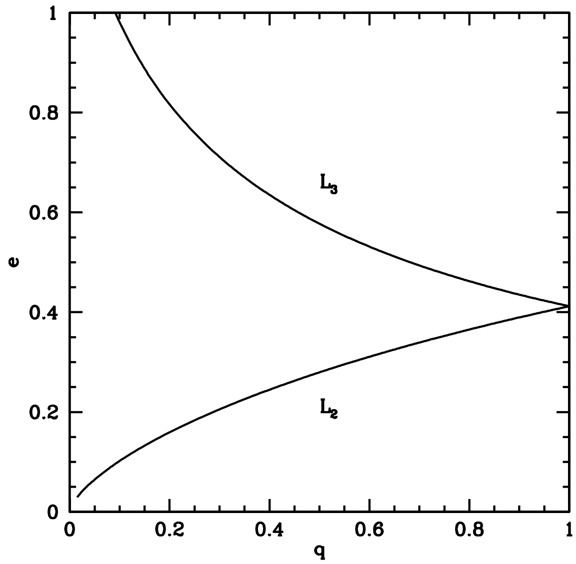

Figure 5: The eccentricity at which the potential at the Lagrangian points L_2 and L_3 is equal to that at L_1 as a function of mass ratio q. For eccentricities greater than these a growing star will lose matter from its far side before transferring mass directly to its companion.

where ρ and p are the density and pressure, $\phi_{\rm G}$ is the gravitational potential of the stars and

$$\phi_\Omega = -\frac{1}{2}\Omega_{\rm p}^2 \left\{ \left(x - \frac{M_2}{M_1+M_2} r_{\rm p} \right)^2 + y^2 \right\} \tag{7}$$

is the centrifugal potential, where x and y are Cartesian coordinates in the plane of the orbit with x in the direction to star 2. Again arguing that star 1 is stationary in this frame we can expect its surface (of constant (external) pressure) to lie on an equipotential surface of

$$\Phi_{\rm T} = -\frac{GM_1}{(x^2+y^2+z^2)^{1/2}} - \frac{GM_2}{((x-r_{\rm p})^2+y^2+z^2)^{1/2}} - \frac{1}{2}\Omega_{\rm p}^2[(x - \frac{M_2 r_{\rm p}}{M_1+M_2})^2 + y^2]. \tag{8}$$

Potential maxima $\partial\Phi_{\rm T}/\partial x = 0$ for real x along $y=0$ and $z=0$ give $x_{\rm L_{1,2,3}}$ and two other Lagrangian points $\rm L_4$ and $\rm L_5$ are located in the orbital plane away from the x-axis. If $e=0$ and $r_{\rm p}=a$ then $\Phi_{\rm T}$ reduces to the standard Roche potential which we plot for a mass ratio $q = M_1/M_2 = 0.5$ in Figure 1. Note the normal features: the potential surfaces tend to spheres close to the stars and far from the system; the $\rm L_2$ and $\rm L_3$ points are at higher potential than the $\rm L_1$.

The sequence of Figures 2 – 4 illustrate the effects of increasing e while keeping q constant. At $e = 0.2$ (Figure 2) the potential surfaces qualitatively resemble those of the standard Roche potential. At $e = 0.4$ (Figure 3) the surface through $\rm L_2$ has a lower potential than that through $\rm L_1$. In this case if star 1 expands to fill this surface at periastron we would expect it to shed material through the $\rm L_2$ point. If this material were always forced to corotate at $\Omega_{\rm p}$ it would be expelled from the system. However it is not and may fall back either on to star 1 itself or accrete on to the companion. The surface through $\rm L_1$ does not lie outside that through $\rm L_2$ so that if the star were a little larger it would lose material both at L_2 and L_1 with that passing through L_1 most likely being accreted by star 2. At $e = 0.8$ (Figure 4) both stars have potential surfaces opening to the outside and therefore lose mass through the L_2 and L_3 points. The critical eccentricity at which the potentials at L_2 and L_3 first exceed that at L_1 is plotted in Figure 5 as a function of q.

4 Smoothed-Particle-Hydrodynamic Simulations

We have analysed only the situation at periastron and even that case only approximately. To date no full analytic solution has been found when we include $\ddot{\mathbf{r}}$ in Equation (6) and, in most cases, if a star is unstable at periastron it is likely that mass transfer takes place over a range of orbital phase. To better analyse the true situation we turn to SPH.

Smoothed Particle Hydrodynamics is a Lagrangian particle-based method that has been widely used to tackle all kinds of astrophysical problems. For a description of the method see the reviews by Benz (1990) and Monaghan (1992). We used a version of the SPH code that corresponds to the description in Benz (1990). It has been used to simulate collisions of main sequence stars (Freitag & Benz 2004), merging of neutron stars (Rosswog et al. 1999, Rosswog & Davies 2002), red giant collisions in globular clusters (Davies et al. 1991) and in the Galactic centre (Bailey & Davies 1999), formation of binary stars (Bonnell et al. 1991) and planets (Alibert et al. 2004).

Stars are notoriously difficult to model adequately particularly in their surface layers, that interest us most here, where the pressure scale height is relatively short. Here we model the donor, star 1, as an $n = 3/2$ polytrope formed from 10,000 SPH particles of various masses to strongly enhance mass resolution at the surface. We model the accretor, star 2, as a point mass but do not try to model the accretion itself. This is a good representation of a low-mass X-ray binary that might have recently formed by capture in a globular cluster.

We set up binary systems each with $M_1 = 1\,M_\odot$, $R = 0.89\,R_\odot$, $M_2 = 2\,M_\odot$ and $a = 5.55\,R_\odot$. This is such that Equation 1 would give $R = R_\mathrm{L}$ for $e = 0.5$ if a were replaced by r_p. With these parameters we run models with $e = 0.4$, 0.5, 0.6 and 0.7. We began each simulation at apastron. The $e = 0.4$ case does not noticeably interact and there is no mass loss. In the $e = 0.5$ case a clear tidal Roche-lobe-like distortion forms at periastron and a number of particles flow through the pseudo-L_1 point. We have applied the method of Rasio & Shapiro (1991) to estimate the amounts of mass bound to the stars and lost from the system. After 4 orbits about $2 \times 10^{-3} M_\odot$ is lost, half of which is bound to the companion and half leaves the system. Figure 6 illustrates most of an orbit. The eccentricity $e = 0.5$ marks the onset of mass transfer at periastron for $q = 0.5$ and $\omega = \Omega_\mathrm{p}$. At $e = 0.6$ (Figure 7) there is a clear episode of mass transfer during periastron passage and some mass leaves star 1 through the L_2 point but falls back relatively quickly. About 5% of the donor mass is lost at the first periastron passage. Initially most ($\sim 90\%$) of this lost gas is bound to the accretor but this would decrease with time as the companion's velocity changes direction. An accretion disc-like feature appears to form around star 2.

At $e = 0.7$ (Figure 8) there is a much more dramatic mass transfer at periastron and a fraction

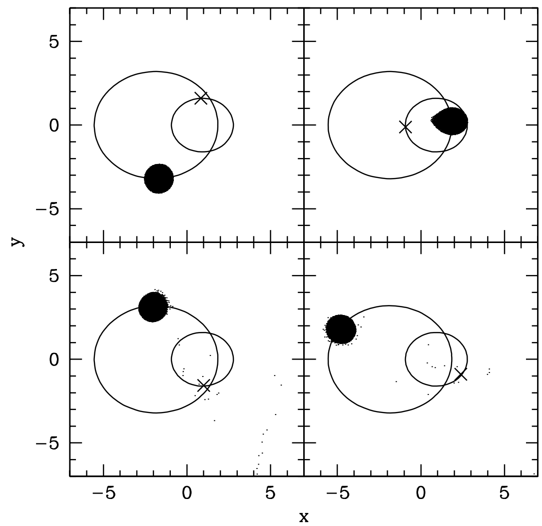

Figure 6: A projection on to the orbital plane of a section of a 10,000-particle SPH simulation of an orbit of a binary system with $q = 0.5$ and $e = 0.5$. A tidal distortion forms at periastron but very little mass is transferred. The cross marks the location of the companion star 2.

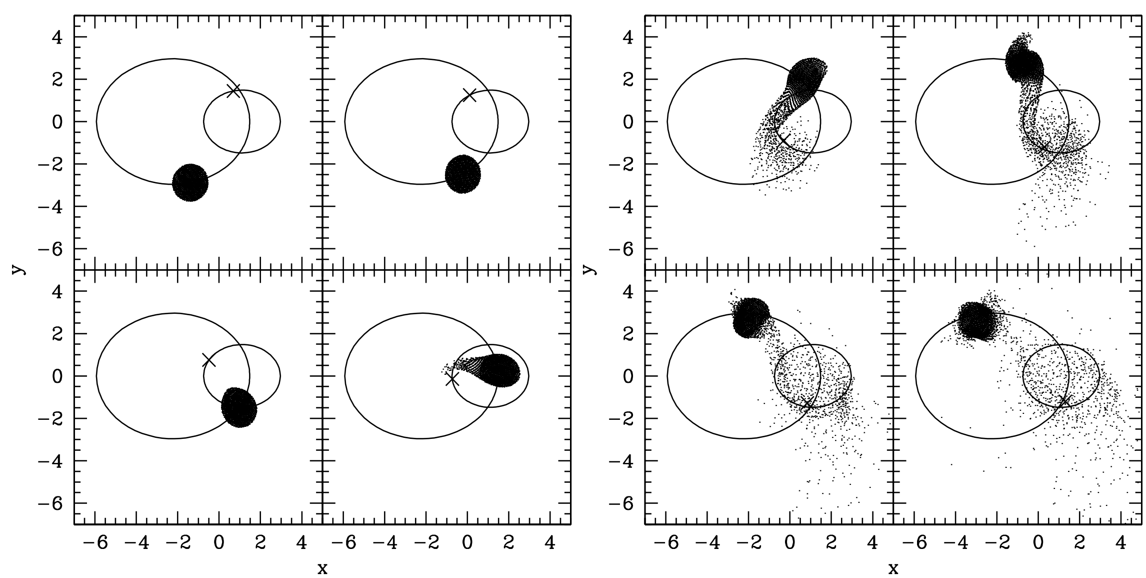

Figure 7: (a - left) As Figure 6 but for the early part of an orbit of a system with $e = 0.6$. (b - right) Continuation of the orbit begun in (a). An accretion disc-like feature appears to form around star 2.

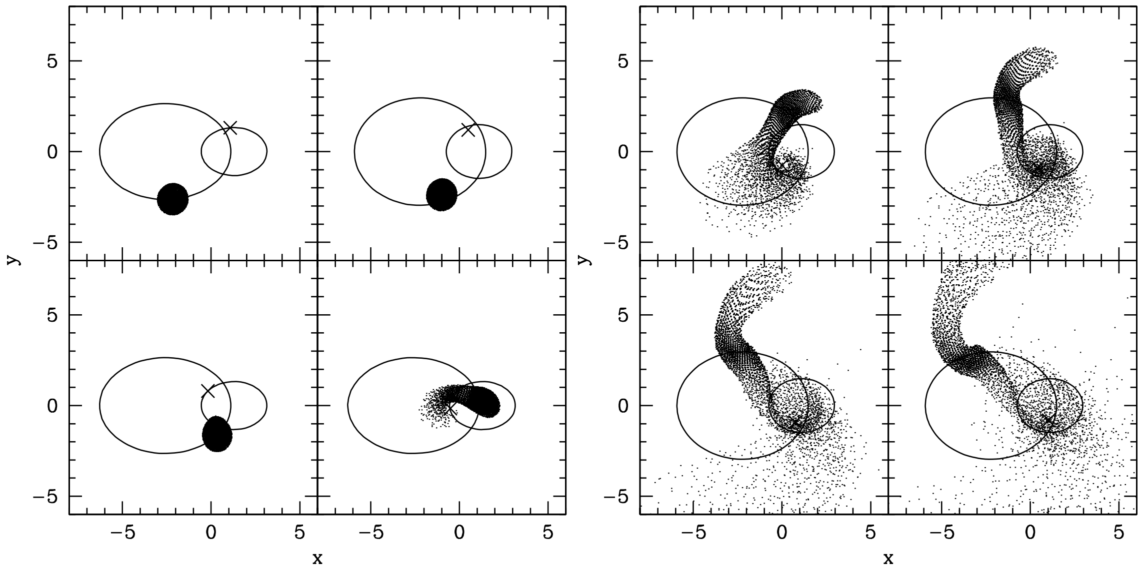

Figure 8: (a - left) As Figure 7 a but for $e = 0.7$. (b - right) Continuation of the orbit begun in (a)

of particles leaving at L_2 do not remain bound to star 1. They will be lost from the system or become bound to star 2. Unfortunately the SPH time steps become too small to complete even a single orbit for this system. Slightly after periastron about 80% of its mass is energetically unbound from the donor but this fraction decreases as the separation increases with the orbital phase. At the end of the simulation 40% of the donor mass has been lost. About 90% of this is bound to star 2.

Figure 9 shows the surface density perpendicular to the orbital plane around star 2 at the end of the $e = 0.7$ simulation. As expected the material appears to be forming an accretion disc. However the modelling of the accretion disc is beyond the scope of this work. The time sequence in Figure 10 illustrates the evolution of the surface density in the orbital plane in the $e = 0.7$ simulation.

5 Conclusions

The predictions of our simple analytic approximation for eccentric binaries are largely justified by the SPH simulations. Our results indicate that the radius of the Roche lobe is similar to circular binaries when calculated for the separation and angular velocity at periastron. Some mass is lost through the (pseudo-) Lagrangian point L_1 from the primary near periastron. However, rather than being all accreted by the secondary some of the stream appears to leave the system. For larger eccentricities part of the mass loss occurs through the L_2 point as the potential at L_2 and L_3 reaches that at L_1 for a given mass ratio q. A more comprehensive set of calculations are required to quantitatively estimate $R_{\mathrm{L}}(q, e, \omega)$ and the mass transfer rate.

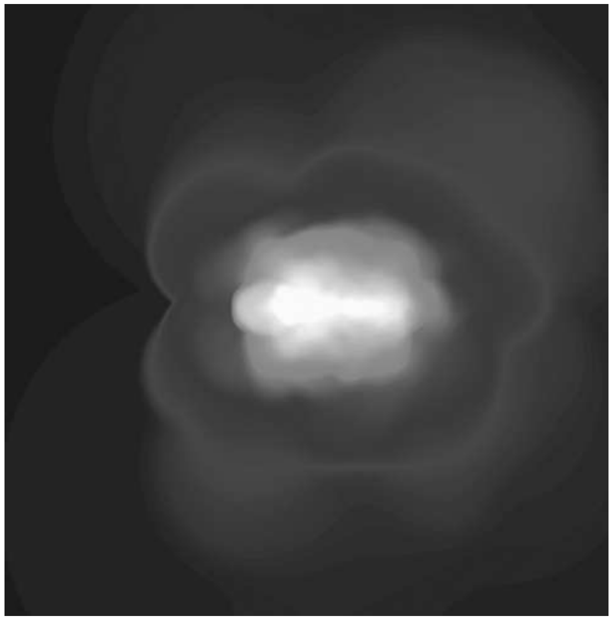

Figure 9: The surface density perpendicular to the orbital plane at the end of the $e = 0.7$ simulation. A disc forms around star 2 on the right.

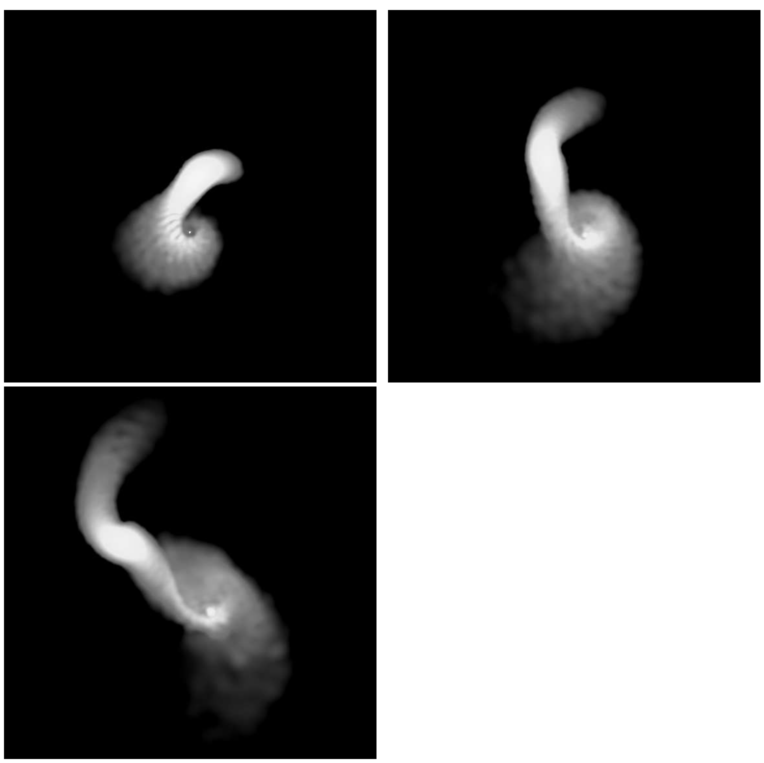

Figure 10: Surface density in the orbital plane in the $e = 0.7$ simulation

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